
UNIVERSITÀ
DI PARMA

PhD Programme in Civil Engineering and Architecture
XXVI Cycle (A.A. 2020/21-2022/23)

# Theoretical Models and Numerical Techniques for the Mechanical Simulation of Soft Matter

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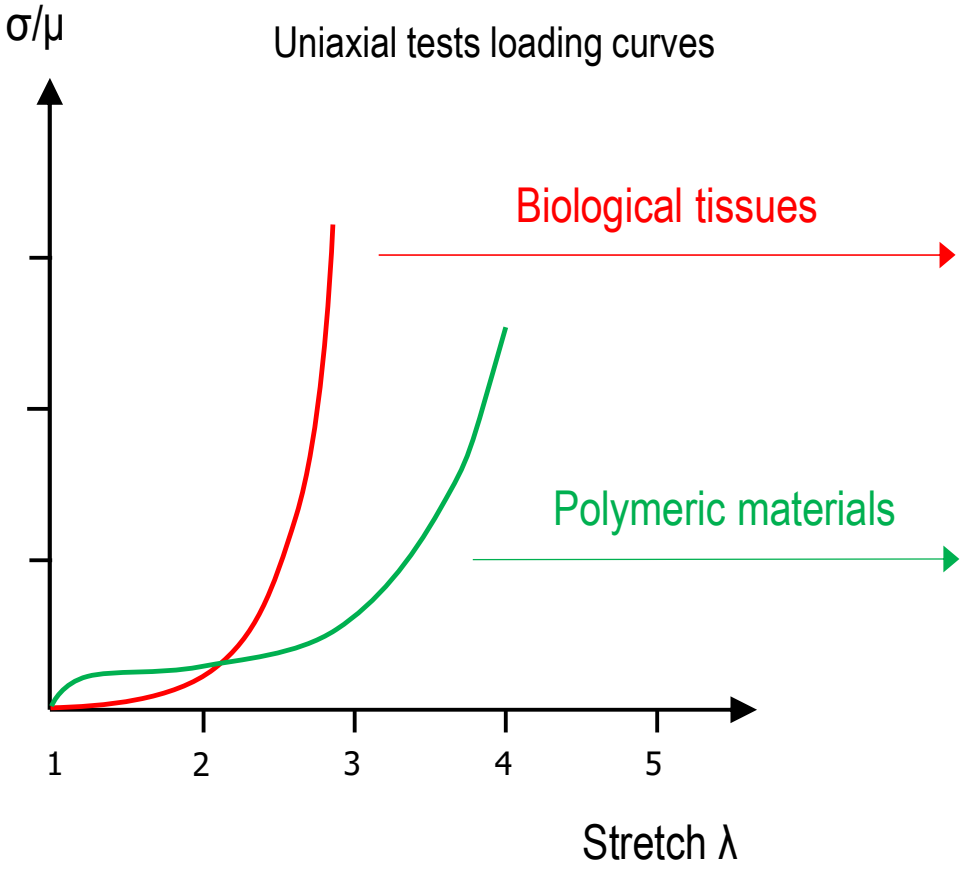
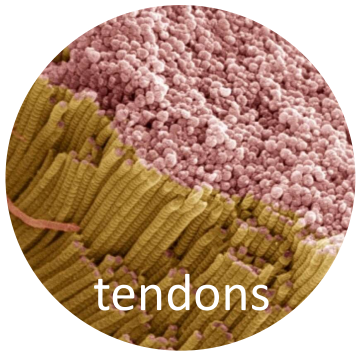
# Soft materials – Hyperelastic behavior



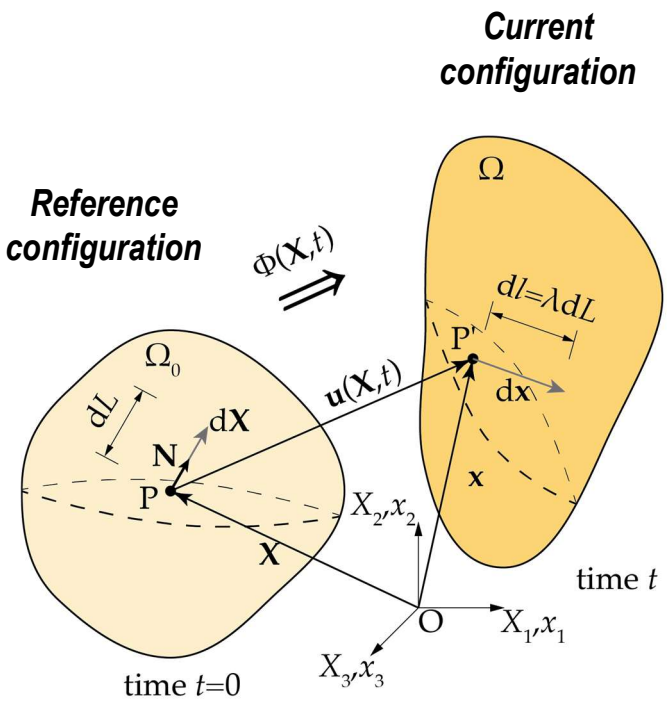






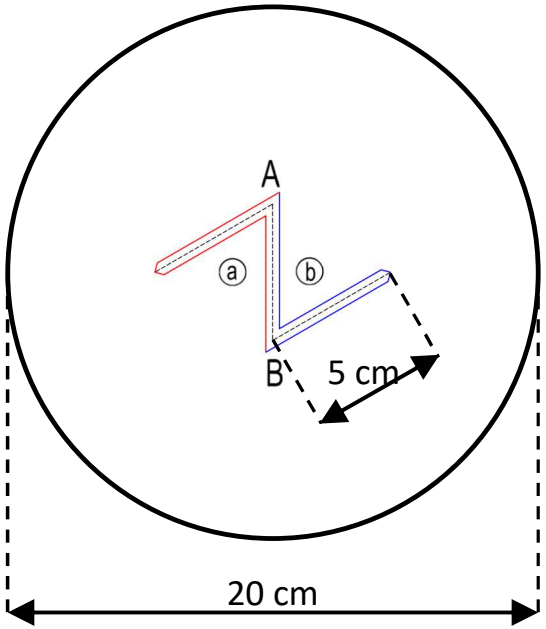


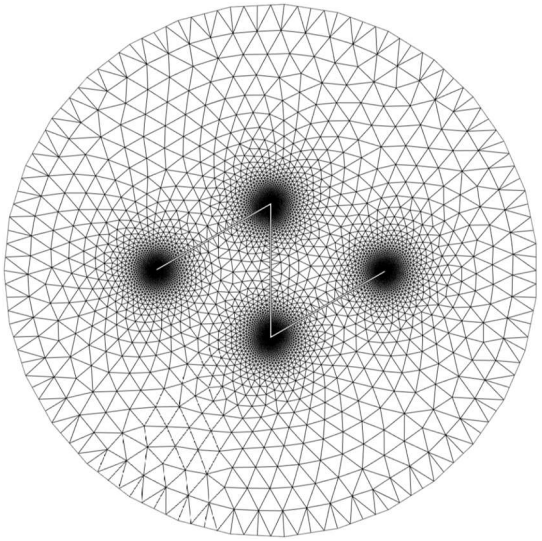
# Analitical instruments for the analysis of non linear elastic materials: Finite strain mechanics

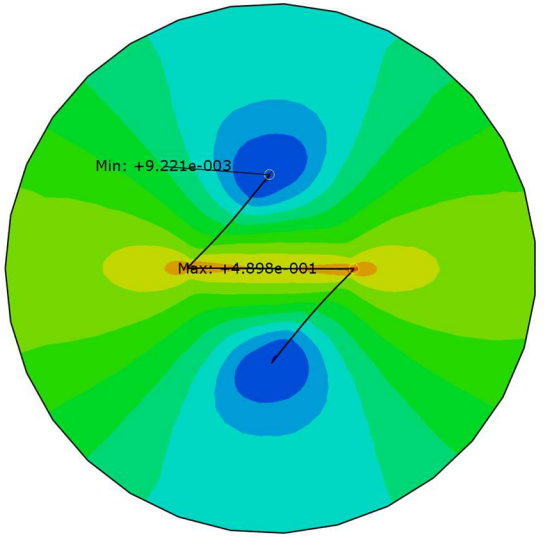



| | |
|---|---|
| Current coordinates | $\mathbf{x} = \Phi(\mathbf{X}, t)$ |
| Current coordinates variation | $\mathrm{d}\mathbf{x} = \mathbf{F}\mathrm{d}\mathbf{X}$ |
| Deformation gradient | $\mathbf{F} = \dfrac{\partial \Phi(\mathbf{X}, t)}{\partial \mathbf{X}}$ |
| Volume | $J = \det\mathbf{F} = \lambda_1\lambda_2\lambda_3$ |
| Ogden potential function | $\Psi(\lambda_1, \lambda_2, \lambda_3) = \sum_{i=1}^{N} \dfrac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right)$ |
| Cauchy stress tensor | $\boldsymbol{\sigma} = \sum_{a=1}^{3} J^{-1}\lambda_a \dfrac{\partial \Psi}{\partial \lambda_a} \hat{\boldsymbol{n}}_a \otimes \hat{\boldsymbol{n}}_a$ |

# Simulation of real surgeries through the framework of the Finite Element Method (FEM)






Model definition:

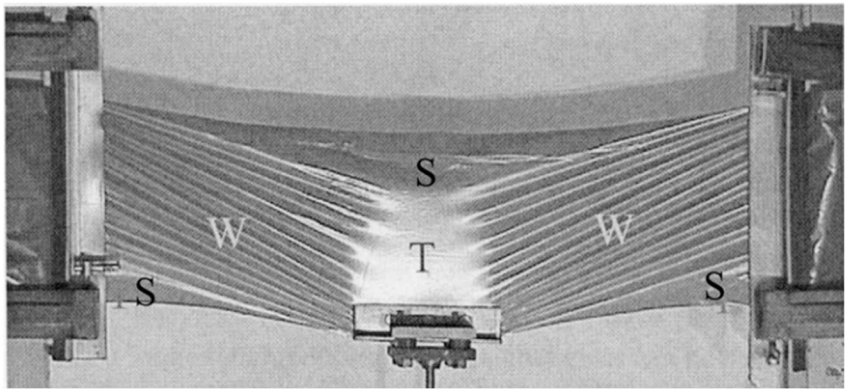
- Z-Plasty Geometry
- Material parameters
- Boundary conditions

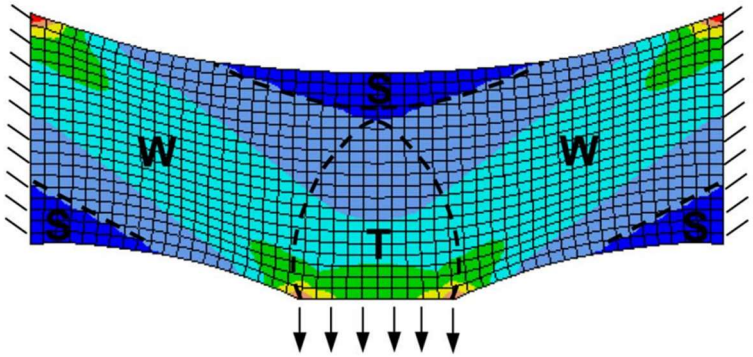
Model creation algorithm:

- Mesh refinement
- Suture multi-point constraint

FEM analysis

# Development of new Constitutive Models: Relaxed Potential Function for Wrinkling Condition in Tensile Uniaxial and Compressive Biaxial Regions




Taut Regions (T)

$$\lambda_1, \lambda_2 \geq 1$$

$$\Psi = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i} \left( \lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + (\lambda_1 \lambda_2)^{-\alpha_i} - 3 \right)$$

Wrinkle Regions (W)

$$\lambda_1 \geq 1$$

$$\lambda_2 < \lambda_1^{-\frac{1}{2}}$$

$$\Psi = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i} \left( \lambda_1^{\alpha_i} + 2\lambda_1^{-\frac{\alpha_i}{2}} - 3 \right)$$

Slack Regions (S)

$$\lambda_1, \lambda_2 < 1$$

$$\Psi = 0$$

(Incompressible plane stress model)